X e Y sono 2 v. a. indipend.

$$P(X=-1)=\frac{1}{5}\ ;\quad P(X=1)=\frac{4}{5}$$

$$P(Y=2)=\frac{1}{5}\ ;\quad P(Y=0)=P(Y=-1)=\frac{2}{5}$$

$$U = X \cdot Y$$

(a) Calcolare $Cov(X,U)$

(b) X e U sono indipendenti?

$$Cov(X,U) = \underbrace{E[XU]}_{} - \overbrace{\underbrace{E[X]}_{}\cdot\underbrace{E[U]}_{}}^{=0}$$

$$E[\underbrace{XU}]$$

$$\varphi(x,u) = x \cdot u$$

$$E[\varphi(X,U)] = \sum_{x,u} \varphi(x,u)\, P(X=x, U=u)$$

$$= \sum_{x,u} x \cdot u\, P(X=x, U=u)$$

$$P(X=-1, U=0) =$$

$$= P(X=-1, X \cdot Y=0) =$$

$$= P(X=-1, Y=0) = P(X=-1)\,P(Y=0)$$

$$E[XU] = E[X(XY)] =$$

$$= E[\underbrace{X^2}_{=}\underbrace{Y}_{=}] = \underbrace{E[X^2]}_{=1}\underbrace{E[Y]}_{=0}$$

$$\varphi(x) = x^2$$

$$\psi(y) = y$$

$$Cov(X, U) = \underbrace{E[XU]}_{=0} - E[X]\underbrace{E[U]}_{=0}$$

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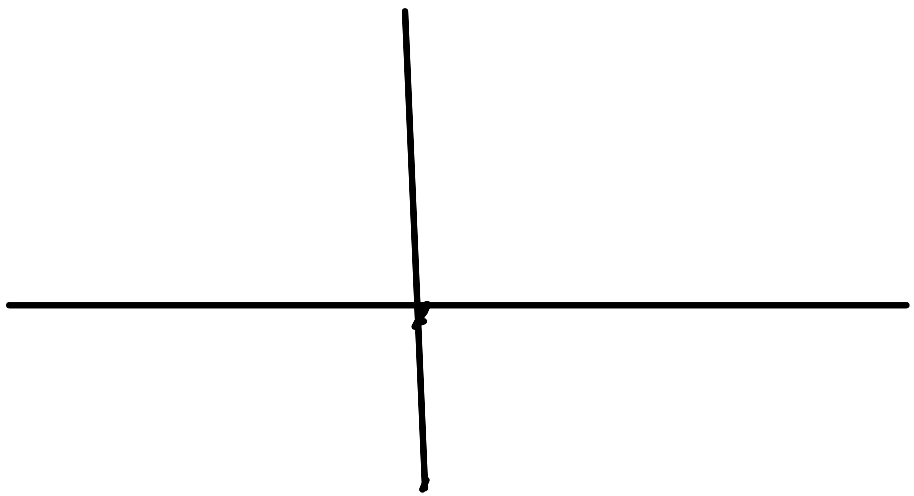
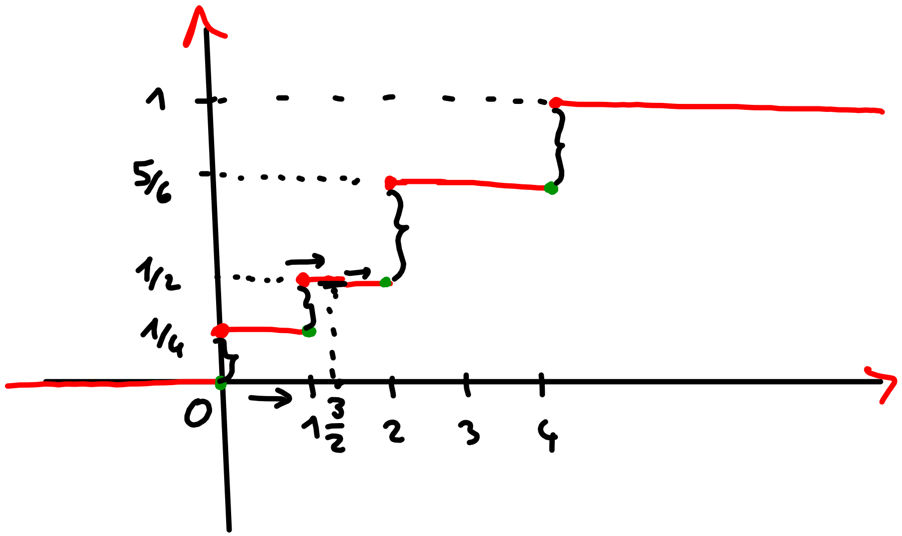
Sia F la funzione

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/4 & 0 \le x < 1 \\ 1/2 & 1 \le x < 2 \\ 5/6 & 2 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$





$0, 1, 2, 4$

$P(X=0)=\frac{1}{4}$; $P(X=1)=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$

$P(X=2)=\frac{5}{6}-\frac{1}{2}=\frac{1}{3}$; $P(X=4)=\frac{1}{6}$

Sia X una v. a. avente f.d.r. $=F$

Calcolare $P\left(1 \le X < \frac{3}{2}\right)$ e $P(X \geq 2)$

$$P\left(1 \le X < \frac{3}{2}\right) =$$

$$P(a \le X < b) = \lim_{x \to b^-} F(x) - \lim_{x \to a^-} F(x)$$

$$= \lim_{x \to \frac{3}{2}^-} F(x) - \lim_{x \to 1^-} F(x) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$= P(X = 1) = 1/4$$

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$$P(X \geq 2) = 1 - \lim_{x \to 2^-} F(x)$$

$$P(X \geq a) = 1 - \lim_{x \to a^-} F(x) = 1 - \frac{1}{2} = \frac{1}{2}$$

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Sia X una v. a. assolutamente continua con densità

$$f(x) = \begin{cases} \frac{1}{2} & -1 < x < 1 \\ 0 & \text{altrove} \end{cases}$$

Sia Y una v. a. discreta tale che

$$P(Y=1) = P(Y=-1) = \frac{1}{2}$$

Sia $U = XY$

(a) Calc. la legge di U (f. d. r.)

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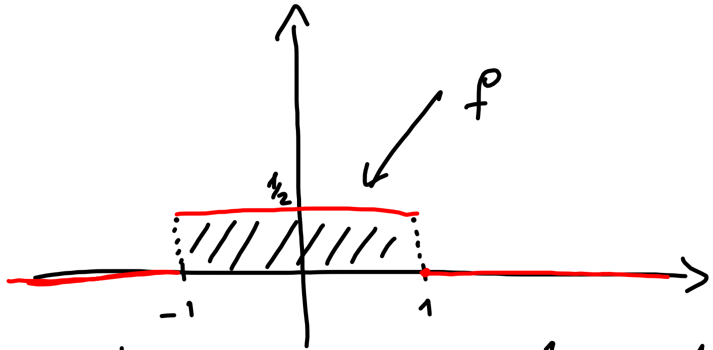
X è ass. cont. con **DENSITÀ** f se $\forall A$ intervallo

$\boxed{f \geq 0}$ e integr. su $\mathbb{R}$

$$P(X \in A) = \int_A f(x)\,dx$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$



$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{-1} 0\,dx + \int_{-1}^{1} \tfrac{1}{2}\,dx + \int_{1}^{+\infty} 0\,dx$$

$$= 0 + \frac{1}{2}\cdot 2 + 0$$

$$= 1$$

$$\underline{\underline{F(t)}} = P(X \leq t) = P(X \in \overbrace{(-\infty, t]}^{A})$$

$$= \int_A f(x)\,dx = \boxed{\int_{-\infty}^{t} f(x)\,dx}$$

$$U = X\,Y$$

$$P(\underline{U \leq t})$$

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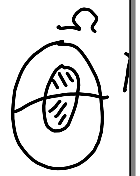
$$\{U \le t\} = \{XY \le t\} =$$

$$\{XY \le t\} \cap \Omega =$$

$$= \{XY \le t\} \cap \left(\{Y=1\} \cup \{Y=-1\} \right)$$

$$= \left(\{XY \le t\} \cap \{Y=1\} \right) \cup$$

$$\cup \left(\{XY \le t\} \cap \{Y=-1\} \right)$$



$$= \{XY \le t, Y=1\} \cup \{XY \le t, Y=-1\}$$

$$\{X \le t, Y=1\} \cup \{-X \le t, Y=-1\}$$

$$P\left(\{X \le t, Y=1\} \cup \{-X \le t, Y=-1\} \right) =$$

$$= P(X \le t, Y=1) + P(-X \le t, Y=-1)$$

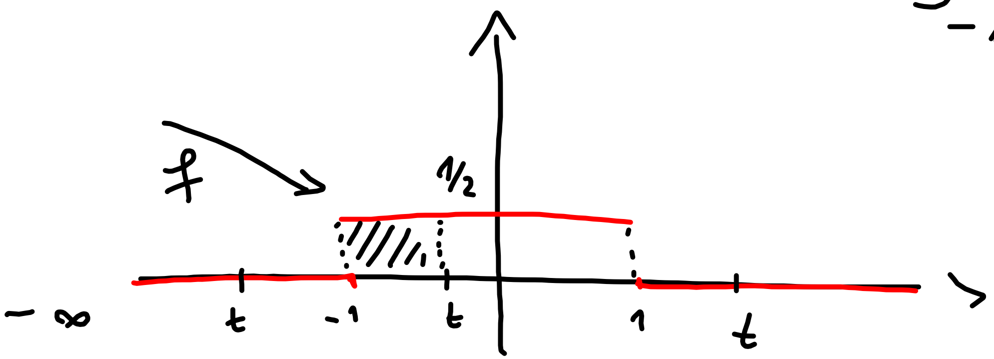
$$= P(X \le t) P(Y=1) + P(-X \le t) P(Y=-1)$$

$$\boxed{P(X \in I, Y \in J) = P(X \in I)\, P(Y \in J) \quad \forall I, J \text{ intervalli}}$$

$$= \frac{1}{2}\,\underline{P(X \le t)} + \frac{1}{2}\,\underline{P(-X \le t)}$$

$$X \sim f : \quad f(x) = \begin{cases} \frac{1}{2} & -1 \le x \le 1 \\ 0 & \end{cases}$$

$$P(X \le t) = \text{f d r. di } X = \int_{-\infty}^{t} f(x)\,dx$$



$$\int_{-\infty}^{t} f(x)\,dx = \begin{cases} 0 & t \le -1 \\ \int_{-\infty}^{-1} + \int_{-1}^{t} = 0 + \frac{t+1}{2} & -1 < t < 1 \\ \int_{-\infty}^{-1} + \int_{-1}^{1} + \int_{1}^{t} = 1 & \end{cases}$$

$$= P(X \le t)$$

$$P(-X \le t) = P(X \ge -t) = P(X \in (-t, +\infty))$$

$$= \int_{-t}^{+\infty} f(x)\,dx = \dots$$

$$= 1 - \lim_{x \to (-t)^-} F(x) = \underline{\underline{1 - F(-t)}}$$

$$\boxed{F(t) = \begin{cases} 0 & t < -1 \\ \frac{t+1}{2} & -1 < t < 1 \\ 1 & t > 1 \end{cases}} \qquad F(-t) = \begin{cases} 0 & \underline{\underline{-t < -1}} \\ \frac{-t+1}{2} & \underline{\underline{-1 < -t <}} \\ 1 & -t > 1 \end{cases}$$

$$= \begin{cases} 0 & t > 1 \\ \frac{1-t}{2} & -1 < t < 1 \\ 1 & t < -1 \end{cases}$$

$$1 - F(-t) = \begin{cases} 1 & t > 1 \\ 1 - \frac{1-t}{2} & -1 < t < 1 \\ 0 & t < -1 \end{cases}$$

$$= \begin{cases} 0 & t < -1 \\ \frac{1+t}{2} & -1 < t < 1 \\ 1 & t > 1 \end{cases} = P(-X \le t)$$

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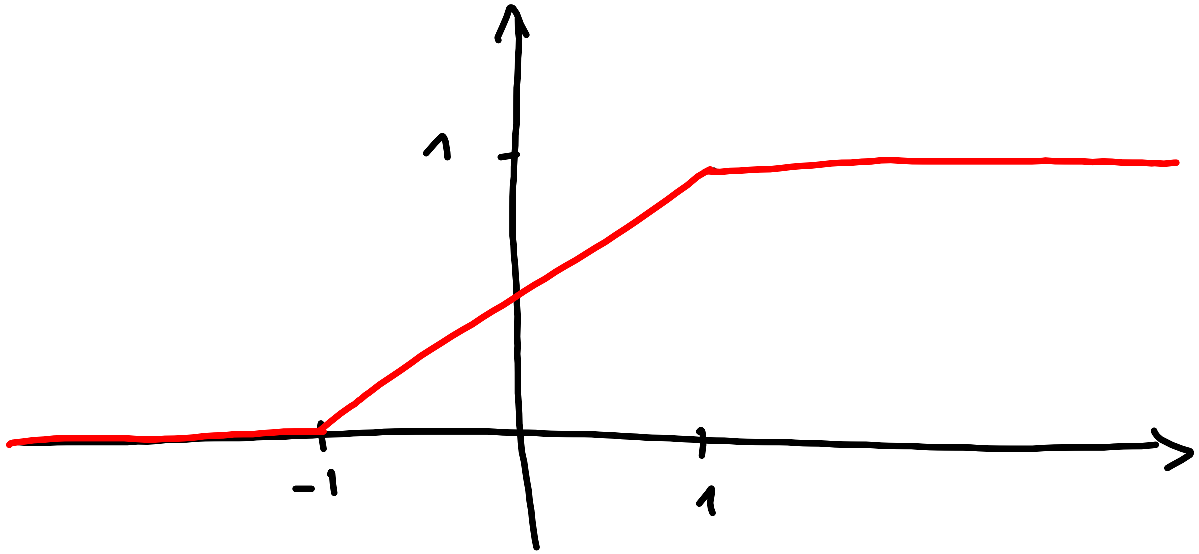

X è continua

$P(X=a)=0$
$\forall a \in \mathbb{R}$

$X \sim f$

$Y = \begin{cases} 1 & 1/2 \\ -1 & 1/2 \end{cases}$ $\qquad 1+Y = \begin{cases} 2 & 1/2 \\ 0 & 1/2 \end{cases}$

$U = XY \sim f$

$Z = X + U$ è una v. a. continua?

$P(Z=a) = 0 \quad \forall a$

$Z = X + XY = X(1+Y)$

$P(\{Z=0\}) = P(\{\underbrace{X=0}_{A}\} \cup \{\underbrace{1+Y=0}_{B}\}) =$

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \underbrace{P(X=0)}_{0} + \underbrace{P(1+Y=0)}_{1/2} - \underbrace{P(X=0, 1+Y=0)}_{0}$

$\{X=0, 1+Y=0\} \subseteq \{X=0\}$

$P(Z=0) = \frac{1}{2}$

Calcolare e disegnare la f.d.r. di Z

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$$X = \begin{cases} -1 & 1/5 \\ 1 & 4/5 \end{cases}$$

$$\underline{\underline{U}} = \underline{X}\,Y$$

$$Y = \begin{cases} 2 & 1/5 \\ 0 & 2/5 \\ -1 & 2/5 \end{cases}$$

$$\varphi(x) = \underline{x} \qquad \psi(x,y) = \underline{x}\,y$$

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$$\underbrace{P(\underline{X=-1}, \underline{U=2})}_{} =$$

$$= P(X=-1, Y=-2) = \underline{0}$$

$$\overbrace{P(X=-1) \cdot P(U=2)} =$$

$$= \frac{1}{5} \cdot \frac{4}{25}$$

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$$P(X=-1, U=2) =$$

$$= P(X=-1, XY=2) =$$

$$= P(\underbrace{X=-1}, \underbrace{Y=-2}) = 0$$

$$\emptyset$$

$$E[X] = \sum x \, P(X=x) =$$
$$= (-1)\cdot\frac{1}{5} + (1)\cdot\frac{4}{5} = \frac{3}{5}$$

$$E[U] = \sum y \, P(U=y)$$

$$X = \begin{cases} -1 \\ 1 \end{cases} \qquad Y = \begin{cases} 2 \\ 0 \\ -1 \end{cases}$$

$$E[U] = -\frac{2}{25} + \frac{8}{25} + \frac{2}{25} - \frac{8}{25} = 0$$

$$U = \begin{cases} -2 & \frac{1}{25} \\ 2 & \frac{4}{25} \\ 0 & 2/5 \\ 1 & 2/25 \\ -1 & 8/25 \end{cases}$$

$$\overline{1}$$

$P(U=-2)$

$$\underbrace{P(A)}_{3} \underset{+}{\cup} \underbrace{P(B)}_{8}$$

$3 \cup 8$

$U = \cdot$

$P(A \cup B)$

$$\{U=-2\} = \{X=-1, Y=2\}$$

$$P(U=-2) = P(X=-1, Y=2) =$$
$$= P(X=-1)\,P(Y=2) =$$
$$= \frac{1}{5}\cdot\frac{1}{5} = \frac{1}{25}$$

$$\{U=0\} = \underbrace{\{\boxed{X=-1}, Y=0\} \cup \{\boxed{X=1}, Y=0\}}$$

$$P(U=0) = P(\cdots) = P(X=-1, Y=0) +$$

$$+ P(X=1, Y=0) =$$

$$= P(X=-1)P(Y=0) + P(X=1)P(Y=0)$$

$$= \frac{1}{5}\cdot\frac{2}{5} + \frac{4}{5}\cdot\frac{2}{5} = \frac{2}{5}$$

$$U = XY$$

$$\{U=0\} = \{Y=0\}$$

$$E[U] = E[XY] \overset{\text{indip. di } X \text{ e } Y}{=} \underbrace{E[X]}\cdot \underbrace{E[Y]}_{=0}$$

$$E[Y] = 2\cdot\frac{1}{5} + (-1)\cdot\frac{2}{5} = 0$$

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